# Laser self-focusing in the presence of quasistatic axial direct current

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We try to understand the presence of a quasistatic magnetic field on the basis of the stability of the laser-plasma system. A general theoretical model of laser self-focusing in the absence of a quasistatic magnetic field (QMF) is extended to discuss self-focusing in the presence of a QMF. Various transverse intensity profiles under different axial collective electronic speeds $V_z$ are calculated. Numerical results indicate that for suitable laser power and plasma density, the increment in $V_z$ can lead to a further separation between the photon fluid and the electron fluid and hence a decrement in the energy of the laser-plasma system. This causes it to be possible for the system state without a QMF, or $V_z=0$ state, to be not stable relative to some $V_z \neq 0$ states.

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## I. INTRODUCTION

Many experiments and numerical simulations [1–8] indicate that a quasistatic circular magnetic field can be generated from the interaction of a strong laser pulse with plasmas. How to understand this phenomenon has become a focus of research. Some researchers [4,6] have realized that the high axial electron current associated with this quasistatic magnetic field is not a result of plasma-wave acceleration [9]. There exist some theoretical investigations [10–13] addressing the direct generation of dc magnetic fields from laser-plasma interactions. In the framework of cold fluid theory, a perturbation analysis of the generation of dc magnetic fields from the interaction of a laser with large-longitudinal-gradient plasma is presented in Ref. [12]. Although the perturbation theory in Ref. [12] is successful in elucidating the relevant physics behind this phenomenon, it is necessary for us to develop a theory about this issue beyond the perturbation approximation if we try to completely understand this phenomenon. In the perturbative analysis [12], all physical quantities are expanded as a power series of the perturbation parameter $\varepsilon$, and the ratio between the dc physical quantities and the oscillating physical quantities is assumed to be $\varepsilon$. Strictly speaking, one cannot determine the value of $\varepsilon$ merely from perturbation analysis itself. Obviously, when the perturbation parameter is zero ($\varepsilon=0$), no dc physical quantity appears. Hence, a fundamental question remains to be answered, why in some cases the laser-plasma system can be described by a nonzero perturbation parameter $\varepsilon \neq 0$ (i.e., the dc physical quantity appears) and in other cases be described by $\varepsilon=0$ (i.e., the dc physical quantity disappears). For convenience, we denote the laser-plasma system as an ac state when no dc physical quantity appears and as a non-ac state when the dc physical quantity appears.

In present theoretical models of laser self-focusing [14–19], plasma is taken as unmagnetized. Under this assumption, a plasma electron only quivers transversely in a laser field; i.e., its momentum only includes a transverse component $p_\perp = eA$. Here, $A$ is the laser vector potential. The presence of a longitudinal current means that an electron has a nonzero longitudinal component $p_\parallel(V_z)$ in addition to $p_\perp$. Why the electronic momentum is $p=p_\perp+p_\parallel$ rather than $p=p_\perp$ deserves investigation. A variation in $V_z$ can change the laser transverse structure $I=|A|^2$ because of the relativistic mass associated with $V_z$. This relation between $V_z$ and $|A|^2$ drives us to consider the stability of the laser-plasma system when the laser is strong. If the system keeps itself in the $V_z=0$ state, the plasma fluid does not have axial flowing and the laser keeps itself in a transverse self-focusing structure in an unmagnetized plasma. Here, the axial drifting speed of the electron fluid $V_z$ is a parameter to identify the state of the laser-plasma system. In principle, the system can be in any $V_z$ state. The state with the largest possibility of being present corresponds to the state most frequently observed in experiment. The observation of a quasistatic magnetic field requires us to investigate the stability of a $V_z \neq 0$ state relative to the $V_z=0$ state. We will study in what condition the system has a larger possibility to stay at a $V_z \neq 0$ state than at a $V_z=0$ state. In other words, the presence of quasistatic axial current is understood as a transition of the system state, and the condition for such a transition occurring is the purpose of this work. Here, the condition refers to the system parameters such as laser power and plasma density. Whether or not this phenomenon always takes place under any values of the laser power and of the plasma density is an attractive problem. If not, we should explain why the occurrence of this phenomenon depends on this condition. The stability analysis might lead to a valid explanation of such a conditional occurrence.

This work is organized as follow: the basic equations are given in Sec. II. The calculation results and the comparison with experimental results are given in Sec. III. Section IV is a short summary.

## II. THEORY

### A. Equation of the beam structure

Newton's equation and the continuity equation describe the response of plasmas to a laser field [14]:

$$\partial_t p_z = - m_e c^2 \partial_z(\gamma_\perp \gamma_z) + \partial_z e\phi, \tag{1}$$

$$\partial_t(p_\perp - eA) = - m_e c^2 \partial_\perp(\gamma_\perp \gamma_z) + \partial_\perp e\phi, \tag{2}$$

 

$$\partial_t n + \nabla \cdot \left( \frac{p}{\gamma_\perp \gamma_z m_e} n \right) = 0. \quad (3)$$

Here, $\phi$ is the charge separation potential between electrons and ions. $\gamma_\perp = \sqrt{1+(e/mc^2)^2 I}$ and $\gamma_z = 1/\sqrt{1-(V_z/c)^2}$ are the transverse and longitudinal relativistic factors, respectively. $I = |a|^2$ is the laser intensity and $a$ is the laser vector potential. $n$ is the density of electrons and $m_e$ is the rest mass of electrons. $p_\perp$ and $p_z$ are the transverse and longitudinal momenta respectively. Obviously, over a laser cycle, $p_\perp = eA$ and $p_z = \text{const}$ is a possible response in the electronic fluid velocity when the intensity profile $I$ does not vary in axial direction. The cavitation effect is indicated by the electronic fluid density: $n_e = \min(N_i + m_e c^4 / 4\pi e^2 \nabla^2 \gamma, 0)$. Here $N_i$ is the ionic density.

We consider an $S$-polarized incident laser of the form

$$A^{lab} = a(z_0, \xi, t) \exp[ikz_0 - i\omega t], \quad (4)$$

$$z = z_0 + \xi; kc = \sqrt{\omega^2 - \omega_p^2, 0}, \quad (5)$$

where $A^{lab}$ denotes the vector potential in the laboratory frame. $z$ is the longitudinal coordinate in the laboratory frame. $z_0$ is the longitudinal coordinate of the pulse center in the laboratory frame, and $-l < \xi < l$ represents the relative position to the pulse center. $\omega$ and $k_0$ are laser frequency and wave vector, respectively. $\omega_{p,0}$ is the plasma frequency. First, we consider the case in which the longitudinal speed of the electronic fluid is zero. The equations describing this electromagnetic field, in the $V_z = 0$ case, read

$$[c^2 \nabla^2 - \partial_{tt}] A = \frac{4\pi}{c} j_\perp, \quad (6)$$

$$j_\perp = e n_e \frac{eA}{\gamma m_e c}, \quad (7)$$

where $j_\perp$ is the current density associated with the transverse quiver motion, and $\nabla^2 = \nabla_\perp^2 = \frac{1}{4}(\partial_{z_0 z_0} + 2\partial_{z_0}\partial_\xi + \partial_{\xi\xi})$ Using Eqs. (6) and (2), we derive a nonlinear Schödinger equation [14] of $a$:

$$i\partial_t a = -\frac{1}{2\omega}\left( c^2 \nabla_\perp^2 a + \frac{c^2}{4}\partial_{\xi\xi} a - \partial_{tt} a \right) - \frac{1}{4}\frac{ikc^2}{\omega}\partial_\xi a + \frac{1}{2\omega} V(a) a, \quad (8)$$

$$V = \frac{\omega_{p,0}^2}{\gamma} - \omega^2 + \frac{1}{4}k^2 c^2 + \left[ \frac{c^2 \nabla_\perp^2 \gamma - \partial_{tt}\gamma + \frac{c^2}{4}(\partial_{z_0 z_0}\gamma + 2\partial_{z_0}\partial_\xi \gamma + \partial_{\xi\xi}\gamma)}{\gamma} \right], \quad (9)$$

where $a$ has been scaled to $e/mc^2$ and, correspondingly, the action $\mathfrak{A}$, Lagrangian, and Hamiltonian associated with this Schrodinger equation read

$$\mathfrak{A} = \text{Re}\, \mathcal{A} + \text{Im}\, \mathcal{A}, \quad (10)$$

$$\text{Im}\, \mathcal{A} = \int \left\{ i\frac{1}{2}\partial_t I + ic^2 \frac{k}{8\omega}\partial_\xi I \right\} d\tau dt, \quad (11)$$

$$\text{Re}\mathcal{A} = \int \left( \int \left\{ I\left[ \frac{1}{2\omega}(\partial_t \theta)^2 - \partial_t \theta \right] + \frac{1}{2\omega}(\partial_t \theta)^2 \left( \frac{1}{4I} - \frac{1}{4(1+I)} \right) \right\} d\tau - H \right) dt, \quad (12)$$

$$H = \frac{1}{2\omega}\int \left\{ \left[ \frac{k\partial_\xi \theta}{2} + (\nabla_\perp \theta)^2 + \left( \frac{\partial_\xi \theta}{2} \right)^2 + \left( \frac{k}{2} \right)^2 \right] Ic^2 + \frac{2\omega_{p,0}^2}{\gamma_z}\sqrt{1+I} - \omega^2 I + c^2 (\nabla_\perp I)^2 \left( \frac{1}{4I} - \frac{1}{4(1+I)} \right) + c^2 \left( \frac{\partial_\xi I}{2} \right)^2 \left( \frac{1}{4I} - \frac{1}{4(1+I)} \right) \right\} d\tau, \quad (13)$$

where $\theta$ is the phase of $a$, $I = |a|^2$ is the intensity, and $d\tau = dxdyd\zeta$ is the volume unit of the laser pulse.

The continuity equation of the vector potential $a$ can be directly derived from Eq. (8):

$$i\partial_t |a|^2 = \frac{i}{2\omega}\left[ \left( 2\partial_{tt}\theta - 2c^2 \nabla_\perp^2 \theta - \frac{c^2}{2}\partial_{\xi\xi}\theta \right)|a|^2 - 2c^2 \nabla_\perp \theta \nabla_\perp |a|^2 - \frac{c^2}{2}\partial_\xi \theta\, \partial_\xi |a|^2 - \frac{1}{2}kc^2 \partial_\xi |a|^2 + 2\partial_t \theta\, \partial_t |a|^2 \right]. \quad (14)$$

As the phase of $a$, $\theta$, meets the relations

$$\nabla_\perp \theta = 0; \partial_\xi \theta = -k, \quad (15)$$

$$\partial_t \theta = -\mu; \partial_{tt}\theta = 0, \quad (16)$$

the laser intensity is steady or time independent, $\partial_t I = 0$. The Lagrangian and Hamiltonian at those steady intensity profiles are

$$L^s = \frac{1}{2\omega}\int S(\mu) I d\tau - H^s, \quad (17)$$

$$S(\mu) = 2\omega\mu + \mu^2, \quad (18)$$

$$H^s = \frac{1}{2\omega}\int \left[ \frac{2\omega_{p,0}^2}{\gamma_z}\sqrt{1+I} - \omega^2 I + c^2 (\nabla_\perp I)^2 \left( \frac{1}{4I} - \frac{1}{4(1+I)} \right) + \frac{1}{4}c^2 (\partial_\xi I)^2 \left( \frac{1}{4I} - \frac{1}{4(1+I)} \right) \right] d\tau. \quad (19)$$

Now we consider the equations describing the laser field when the electronic fluid has a longitudinal speed $V_z$. Obviously, the transverse current density $j_\perp$ depends on the longitudinal relativistic factor $\gamma_z$. Because $A$ is a vector and the longitudinal current density $j_z$ is perpendicular to $A$, it seems plausible to believe that the variation of the electron mass is only the result of longitudinal fluidity and the right-hand side of the wave equation of $A$ is only the transverse current density $j_\perp$. On the other hand, we know that Eq. (8) [or Eq. (6)] can be obtained from variation principle $\delta L / \delta a^* = 0$ is

$i\partial_t a = \delta H/\delta a^*$ where $H$ is the Hamiltonian describing an electromagnetic field in a medium without longitudinal electronic current. As longitudinal electronic current exists in the medium, the Hamiltonian or energy describing the longitudinal drifting movement of electrons is

$$H_e \int \frac{1}{2}\gamma_\perp \gamma_z n_e m_e V_z^2 d\tau. \tag{20}$$

Considering $v_z^2 = 1-\gamma_z^{-2}$, we rewrite $H_e$ as

$$H_e = \frac{m_e^2 c^4}{4\pi e^2}(\gamma_z - \gamma_z^{-1})\left[\int \frac{1}{2}[\omega_{p,0}^2\sqrt{1+I}]d\tau\right] + \frac{m_e^2 c^4}{4\pi e^2}(\gamma_z^2 - 1) \times\left[\int \frac{1}{2}[\omega_{p,0}^2\sqrt{1+I}c^2\nabla_\perp^2\sqrt{1+I}]d\tau\right]. \tag{21}$$

This Hamiltonian $H_e$ is in the form of kinetic energy of uncharged particles. Actually, it can also be written in the form of the electromagnetic energy associated with charge current, the $j\cdot A$ term. The total Hamiltonian for this laser-plasma system, which is the combination of $H$ and $H_e$ in the same unit, now reads in a dimensionless form

$$H^T \int \left\{\frac{1}{2}\left[\frac{1}{\gamma_z}2\omega_{p,0}^2\sqrt{1+I} - \omega^2 I + c^2(\nabla_\perp I)^2\left(\frac{1}{4I} - \frac{1}{4(1+I)}\right)\right] + \frac{1}{2}\omega_{p,0}^2\sqrt{1+I}(\gamma_z - \gamma_z^{-1}) + \frac{1}{2}\omega_{p,0}^2 c^2\left(-\frac{1}{4}\frac{(\nabla_\perp I)^2}{1+I}\right)(\gamma_z^2-1)\right\}d\tau. \tag{22}$$

If we now apply the variation principle to the total Hamiltonian $H^T$, it is easy to obtain a Schrödinger equation when the longitudinal current exists:

$$i\partial_t a = \frac{\delta H}{\delta a^*} + \frac{\delta H_e}{\delta a^*}. \tag{23}$$

From this equation, we can find that, except the modified transverse current, there is another term $\delta H_e/\delta a^*$ appearing on the right-hand side of this equation. As $V_z=0$ or $\gamma_z=1$, this term disappears and Eq. (23) returns to Eq. (8). Although the longitudinal current density $j_z$ is a vector perpendicular to $A$, we should notice that it depends on the vector $A$. Hence, the longitudinal fluidity has an indirect effect on the transverse vector potential via the term $\delta H_e/\delta a^* = (\delta H_e/\delta I)a$. This new equation depends on the longitudinal electronic speed $V_z$ via relativistic factor $\gamma_z$. Equations (1) and (23) consist of an equation set in which $a$ and $V_z$ are dependent on each other. This equation set is our basic model of the laser-plasma system. The states of the laser-plasma system are described by $a$ and $V_z$.

The equation of the steady intensity profile for $V_z \neq 0$ can be derived from Eq. (23):

$$s+\omega^2 = -\frac{2}{4}\left[\frac{1}{I} - \frac{1}{1+I}\left(1 - \frac{\omega_{p,0}^2}{2}(\gamma_z^2-1)\right)\right]\nabla_\perp^2 I + \frac{2}{4}\left[\frac{1}{I^2} - \frac{1}{(1+I)^2}\left(1-\frac{\omega_{p,0}^2}{2}(\gamma_z^2-1)\right)\right](\nabla_\perp I) + \frac{1}{\sqrt{1+I}}\left(\frac{\omega_{p,0}^2}{2}(\gamma_z+\gamma_z^{-1})\right), \tag{24}$$

or, equivalently,

$$\mathcal{C} = -(s+\omega^2)I + \frac{(\nabla_\perp I)^2}{4}\left[\frac{1}{I} - \frac{1}{1+I}\left(1-\frac{\omega_{p,0}^2}{2}(\gamma_z^2-1)\right)\right] + \sqrt{1+I}[\omega_{p,0}^2(\gamma_z+\gamma_z^{-1})], \tag{25}$$

where the term on the left-hand side is a space-independent constante $\mathcal{C}$, $s=2\omega\mu=\mu^2$, and $\mu=\partial_t\theta$ is the frequency shift of the laser field. Here, we should note that if the evacuation occurs—i.e., $n_e = \min(N_i + (m_e c^4/4\pi e^2)\nabla_\perp^2\gamma, 0) = 0$, the laser field in the evacuation region of $n_e=0$ is described by

$$s+\omega^2 = -\frac{2}{4}\frac{1}{I}\nabla_\perp^2 I. \tag{26}$$

We discuss the laser field of the vector potential $A \sim \exp(i\omega t)$ and fixed laser power

$$P = \int \omega^2|A|^2 2\pi r dr = \int [\omega+\mu(v_z)]^2|A|^2 2\pi r dr. \tag{27}$$

Considering the singularity of the operator $\nabla_\perp^2(1/r)\partial_r(r\partial_r)$ at $r=0$, we have $\partial_r I|_{r=0}=0$. We can solve the laser profile and the laser frequency from Eq. (25). Note that during the calculation we always monitor whether or not evacuation has occurred.

### B. Stability analysis

As shown previously, $a$ of different $v_z$ parameters obeys the respective variational equations

$$i\partial_t a_0 = \left.\frac{\delta H_T}{\delta a^*}\right|_{a=a_0}, \tag{28a}$$

$$i\partial_t a_{v_z} = \left.\frac{\delta H_T}{\delta a^*}\right|_{a=a_{v_z}}. \tag{28b}$$

We can expand $H_T$ and $a$ about $H_T(a=a_0)$ and $(a=a_0)$:

$$H_T(a=a_{v_z}) = H_T(a=a_0) + \Delta H(a_{v_z} - a_0), \tag{29a}$$

$$\Delta a = a_{v_z} - a_0 = a_0\{f_{v_z}\exp[-i\mu(v_z)t] - 1\}, \tag{29b}$$

where the relative amplitude $f_{v_z}$ is real. Thus we can rewrite the Schrödinger equation of $a$ as [where $H_0 = H_T(a=a_0)$]

$$i\partial_t a_0 + i\partial_t(a_{v_z} - a_0) = \left.\frac{\delta H_0}{\delta a^*}\right|_{a=a_0} + \left.\frac{\delta H_0}{\delta \Delta a^*}\right|_{\Delta_{a=a_{v_z}-a_0}} + \left.\frac{\delta \Delta H}{\delta a^*}\right|_{a=a_0} + \left.\frac{\delta \Delta H}{\delta \Delta a^*}\right|_{\Delta_{a=a_{v_z}-a_0}}. \tag{30}$$

Considering the fact $H_0$ is independent of $\Delta a$ and $\Delta H$ is independent of $a_0$, we have

$$\left.\frac{\delta H_0}{\delta \Delta a^*}\right|_{\Delta_{a=a_{ov_z}-a_0}} = 0, \tag{31a}$$

$$\left.\frac{\delta \Delta H}{\delta a^*}\right|_{a=a_0} = 0. \tag{31b}$$

From Eqs. (28), (30), and (31), we obtain an equation

$$i\partial_t(a_{v_z} - a_0) = \left.\frac{\delta \Delta H}{\delta \Delta a^*}\right|_{\Delta_{a=a_{v_z}-a_0}} = \frac{d\Delta H}{dv_z}\frac{dv_z}{d\Delta a^*}, \tag{32}$$

which can be written in a more clear form

$$i\partial_t(a_0\{f \exp[-i\mu(v_z)t] - 1\}) = \frac{d\Delta H}{dv_z} \Bigg/ \left[ it\, f a_0^* \frac{d\mu}{du_z} \exp[i\mu(\mu_z)t] \right], \tag{33a}$$

i.e.,

$$i\partial_t(a_0\{f \exp[-i\mu(v_z)t] - 1\}) = \left[ \frac{1}{t}\frac{d\Delta H}{dv_z}\frac{1}{f a_0^*} \exp[-i\mu(v_z)t] \right] \Bigg/ \frac{d\mu}{dv_z}. \tag{33b}$$

From this equation, we can easily obtain

$$-\partial_t[I_0 f^2] + f\cos(\mu t)\partial_t I_0 = \frac{2}{t}\frac{d\Delta H}{dv_z} \Bigg/ \frac{d\mu}{dv_z}. \tag{34}$$

We take $I_0$ as time independent and hence obtain an equation describing the growth of the relative intensity:

$$\partial_t f^2 = -\frac{1}{I_0}\frac{2}{t}\frac{d\Delta H}{dv_z} \Bigg/ \frac{d\mu}{dv_z}. \tag{35}$$

From this equation, we know that $f^2$ will grow if the system is in a state of $\partial\Delta H/dv_z < 0$. By Eq. (35), we can study, when the system is initially in a state of the vector potential $a_0$, how soon its evolution to another system state of the vector potential $a_{v_z}$ is. Apprantly, if $\partial\Delta H/dv_z|_{v_z=0} < 0$, the system will begin to leave the $v_z=0$ state to a $v_z=v_1 \neq 0$ state. Similarly, if $\partial\Delta H/dv_z|_{v_z=v_1} < 0$, the system will continue to leave the $v_z=v_1 \neq 0$ state to a $v_z=v_2 \neq 0$ state. This process will continue until the system is in a state of $v_z=v_m$ satisfing $\partial\Delta H/dv_z|_{v_z=v_m}=0$.

For a given $P$, the transverse structure $I$ depends on the longitudinal relativistic factor $\gamma_z$. This leads to the total Hamiltonian having two way depending on $V_z$: one is the obvious dependence of $H^T$ on $V_z$ as expressed by Eq. (22), and the other is dependence of $I$ on $V_z$ as expressed by Eq. (24). These two ways cause $H^T$ to have a complicated dependence on $V_z$. Because the stationary states of the laser-plasma system are identified by different $V_z$, the relationship between the total energy of the system and $V_z$ enables us to discuss the relative stability of those stationary states by comparing their respective energy. In the following numerical experiments, we take a two-step calculation. In the first step we calculate the intensity profile at a given $P$ but under different $V_z$. The total Hamiltonian at those solved intensity profiles is calculated in the second step. We try to find the optimal $V_z$ that corresponds to the lowest value of $H^T$ via this two-step calculation.

## III. NUMERICAL RESULTS AND DISCUSSION

### A. Equilibrium states

In the following numerical calculation, we put field vector potential $a$ in units of the Compton potential $e/mc^2$, length in units of the laser wavelength in microns λ, and ion density $N_i$ in units of the critical density $m_{0,e}\omega^2/4\pi e^2$. Here $\omega$ is taken to be 1.

First we calculate the transverse structure $I(r)$ and the density profile $n_e(r)$. Some examples of $I$ profiles and corresponding $n_e$ profiles are given in Fig. 1. Figure 1 indicates that the increment in $\gamma_z$, when other parameters are given, will lead to a further separation between the laser field and the electron fluid. In other words, more photons congregate in the evacuation region and the evacuation region becomes larger.

The grown separation between the photon fluid and the electron fluid implies electrons possessing less quiver energies. As previously mentioned, the separation between two fluids enables the system energy to vary. This can also be understood by the case in which two fluids do not contact; i.e., they are completely separated. In such a case, the system energy is just the summation of the energy of the laser field in vacuum and that of free electrons. Obviously, the system energy in this case differs from $H^T$ greatly. The variation of $H^T$ with respect to $\gamma_z$ is reflected in Fig. 2. From Fig. 2 one can find that for some values of $P$ and $N_i$, $H^T$ decreases from its value at $\gamma_z=1$ to a minimum at $\gamma_z \neq 1$ and then rises. Such an interesting $H^T$-$\gamma_z$ curve can be qualitatively explained in following manner. For suitable values of $P$ and $N_i$, two fluids are not fully separated from each other; the transverse structure at $\gamma_z=1$ can ensure that there are sufficiently more electrons possessing sufficiently large quiver energies. Such a large quiver energy can afford the energy requirement to promote electrons to a higher $\gamma_z$ state. However, this capacity of promoting electrons to a higher $\gamma_z$ state is not infinite. With $\gamma_z$ rising, two fluids become further separated. Thus, the quiver energy available for promoting $\gamma_z$ decreases whereas the energy requirement for this promotion does not decrease. In other words, the increment in $\gamma_z$ depletes the capacity of promoting $\gamma_z$.

The dependence of $\mu$ on $v_z$ is reflected in Fig. 3. The values of parameters in Fig. 3 equal those in Fig. 2. When $P$ is given, larger $N_i$ corresponds to larger $d\mu/dv_z$, which will decrease the growth rate of $f^2$. Although larger $N_i$ corre-

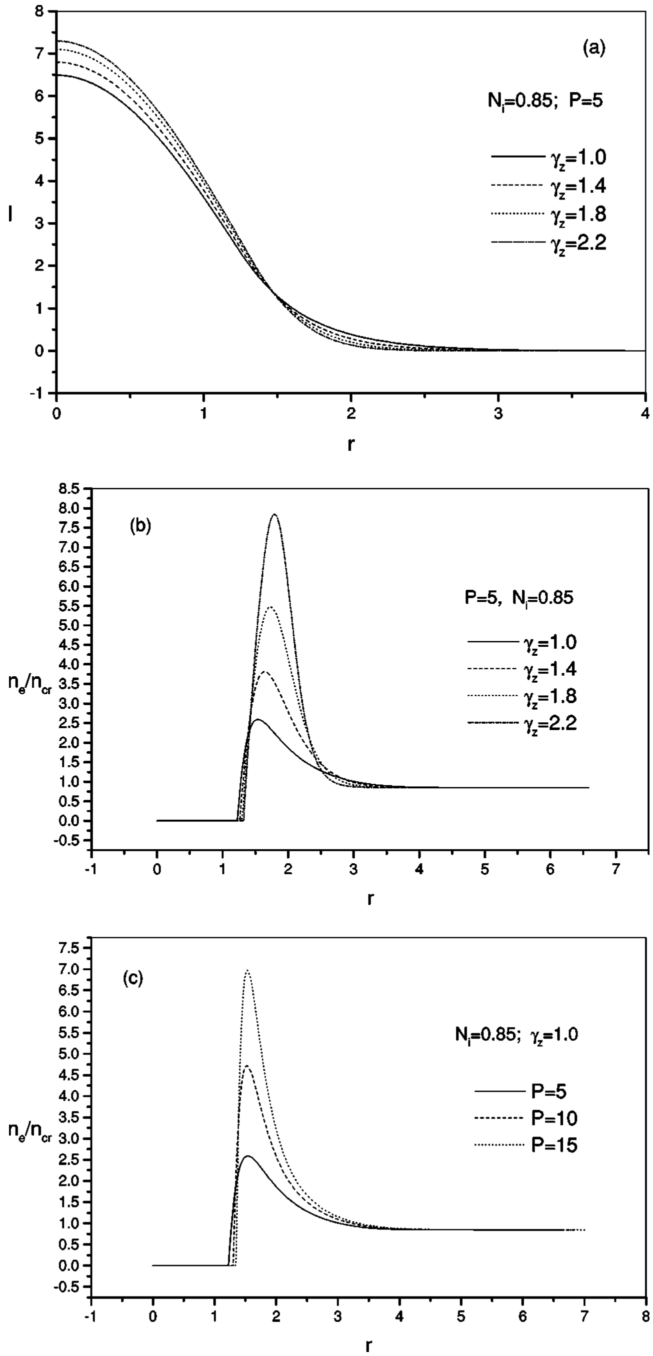


FIG. 1. (a) Tansverse intensity profiles. (b), (c) Density profiles.

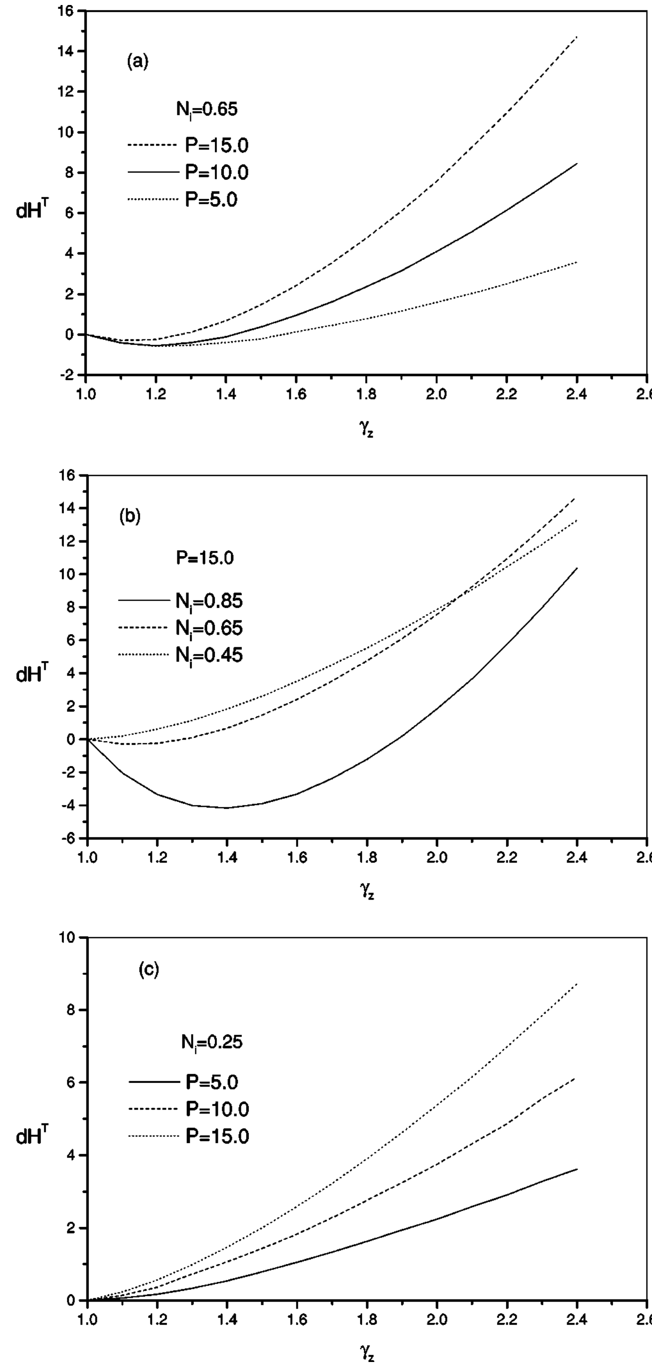


FIG. 2. The system energy vs $\gamma_s$. Here, $dH^T = H^T(\gamma_s, P, N_i) - H^T(1, P, N_i)$.

sponds to larger $|d\Delta H/dv_z|$ ($d\Delta H/dv_z < 0$), it might not mean a faster growth of $f^2$ because of the dependence of $d\mu/dv_z$ on $N_i$. Moreover, the value of $d\mu/dv_z$ is mainly determined by $N_i$. When $N_i$ is given, the variation in $P$ does not cause obvious variations in $d\mu/dv_z$. Because numerical results indicate $d\mu/dv_z > 0$, $f^2$ can grow only when $d\Delta H/dv_z$ becomes negative. Once $d\Delta H/dv_z$ becomes negative, the growth rate also depends on the value of $d\mu/dv_z$. From Figs. 2 and 3, we can find that $d\Delta H/dv_z$ and $d\mu/dv_z$ have a weaker dependence on $P$ but a stronger dependence on $N_i$. This implies that $N_i$ is the primary factor determining whether or not $f^2$ can grow to a substatial level.

### B. Stablest equilibrium state

In the above theory, we have described the state of the laser-plasma system in $V_z$ and $I(r;V_z)$. Obviously, $V_z$ identifies the state of this system. In principle, the system can be in any $V_z$ state. If the system is isolated from its environment, it will stay at its initial state. Here, the environment refers to the plasmas outside the interaction region. In fact, the system can exchange energy with its environment. To some extent, we can view the system and its environment as an isolated ensemble of conserved energy: $E_{ens} = E_{sys} + E_{envir}$. In prin-

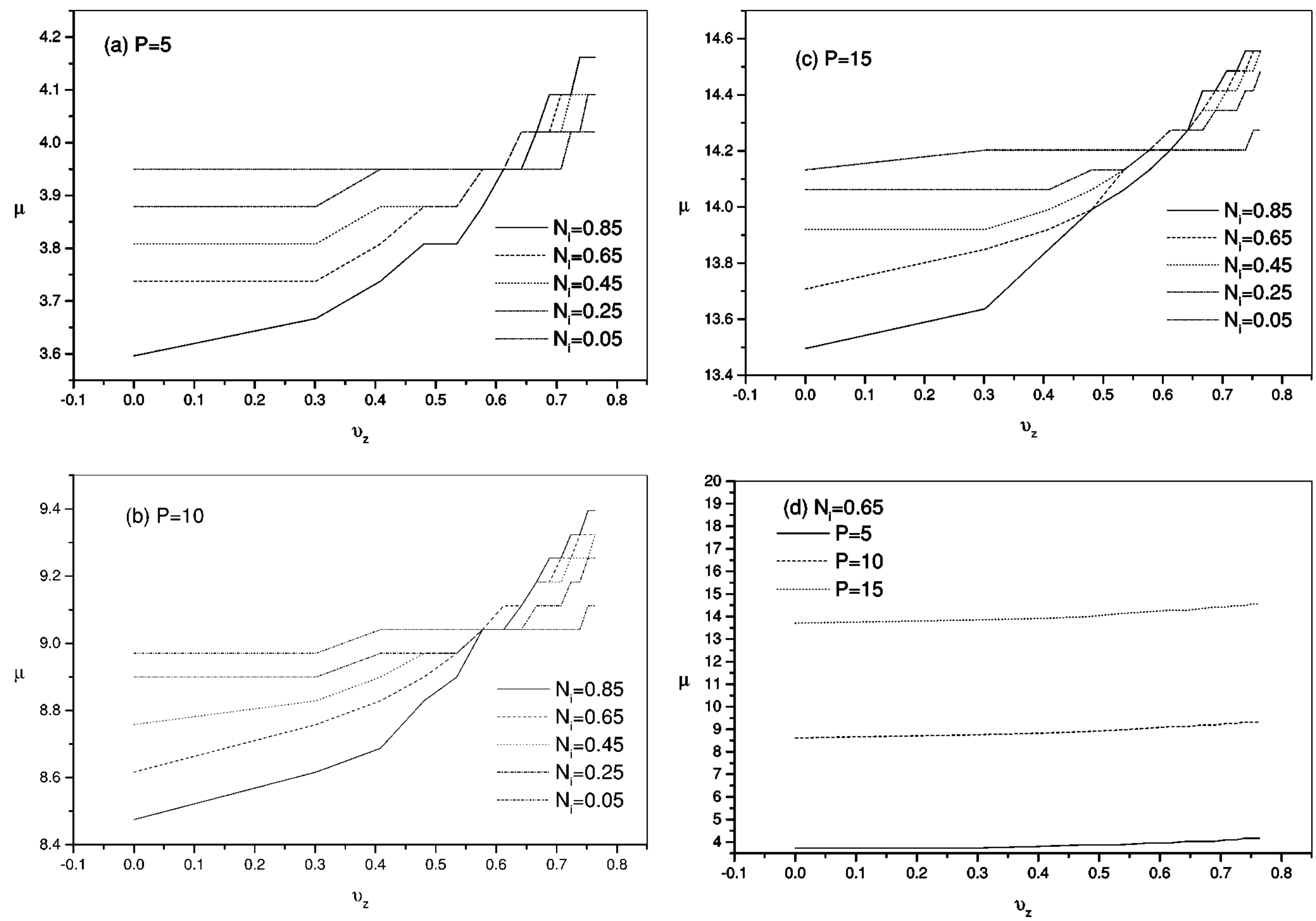


FIG. 3. $\mu$ vs $v_x$.

ciple, this ensemble has equaling possibility to occupy any microscopic state of energy $E_{ens}=E_{sys}+E_{envir}$. Thus for the laser-plasma system, its possibility of occupying a steady state of energy $E_1$ depends on the number of environment microscopic states of energy $E_{ens}-E_1$. For the environment, it consists of numerous noise sources, and its energy can be approximately expressed as the summation of the energy of each noise source, $E_{envir}=\Sigma_j e_j$.

The microscopic state of the environment is described by parameters of noise sources—for example, the energy of every noise source. It should be noted that the larger $E_{envir}$ is, the larger the number of sets $(e_j)$ satisfying $E_{envir}=\Sigma_j e_j$. This suggests a larger $E_{envir}$ corresponding to more microscopic states of the environment. Thus, when the degree of energy exchange between the system and environment is given, smaller $E_1$, or larger $E_{envir}$, is favored.

If a mode does not correspond to a local minimum in $H(V_z)$ curve—i.e., it has a neighboring mode of lower energy—according to the above discussion, it will have less possibility to be occupied than this neighboring mode. In particular, as the energy exchange between the system and environment is extremely weak, this conclusion is still valid. When the system is initially in a mode that is not a local minimum mode, it will leave this initial mode to a mode with lower energy because the initial mode has less possibility to be occupied. From Eq. (35), we know that if negative $d\triangle H/dv_z$ has a sufficiently large value, the growth rate of the laser field transiting from a $V_z=0$ to another $V_z\neq 0$ state is large enough to ensure the transition taking placing in a short time interval, which is characteristic of laser-plasma interactions.

### C. Comparision with experiment

Hence, for suitable $P$ and $N_i$, the stability of the system is possible to require electrons in the interaction region having a nonzero collective speed $V_z$. Once those electrons in the interaction region possess this speed $V_z$, their kinetic energies $\mathcal{E}_z+\mathcal{E}_\perp$ will exceed a $V_z$-dependent threshold $\sim V_z^2/\sqrt{1-(V_z/c)^2}$. Here, $\mathcal{E}_z$ and $\mathcal{E}_\perp$ represent axial and transverse kinetic energies, respectively. In contrast, those electrons outside the interaction region only have zero $\mathcal{E}_z$. This result can be reflected in the distribution $f(E)$. Supposing the distribution in the absence of laser field is Maxwellian and all electrons do not have axial velocity; thus log $f$ is a linear function of $E=\mathcal{E}_\perp$. The presence of a laser field is possible to enable some electrons obtaining a nonzero $\mathcal{E}_z$ while the other not. Because some electrons obtain a positive $\mathcal{E}_z$, correspondingly $f$ decreases in the low-$E$ regime and increases in the high-$E$ regime. Because of an obvious energy difference between electrons inside the interaction region and those outside this region, log $f$ is possible to become two lines jointing. These two lines correspond to electrons outside and inside the interaction region, respectively. Of course, if there is no difference between $\mathcal{E}_z$ inside and $\mathcal{E}_z$ outside the interaction region, log $f$ will remain a straight line.

The electron spectrum reported in Ref. [2] indicates that the energy distribution shows a double-temperature character. Moreover, with the plasma density decreasing, the distribution gradually returns to a single-temperature type. The density-dependent distribution, which transforms from double-temperature type to single-temperature type, suggests that another mechanism, in addition to the wake field acceleration mechanism, might be responsible for the relevant physics. This is because the excitation of plasma wave is valid in a wide density range, especially in the low-density range. From the previous discussion, we find that the return of the distribution to a single-temperature type can be qualitatively accounted for by our theory. As shown in Fig. 3 the $V_z=0$ state, in the low-$N_i$ case, is more stable than other $V_z \neq 0$ state, whereas for the same laser power a $V_z \neq 0$ state is more stable in the high-$N_i$ case, This implies that in the high $N_i$ case, a difference between $\mathcal{E}_z$ inside and $\mathcal{E}_z$ outside the interaction region exists. On the contrary, this difference disappears in the low-$N_i$ case.

## IV. SUMMARY

Extending the model of laser self-focusing in the absence of a quasistatic magnetic field, we discuss the self-focusing structures under different $V_z$. Numerical results indicate that the increment in $V_z$ is possible to strengthen the separation between the photon fluid and the electron fluid and hence lead to the decrement of the system energy. This reveals the laser-plasma system can be at a high-$\gamma_z$ state by increasing the separation between its two ingredient fluids. Some system parameters, laser power and plasma density, are important to determine whether or not the system can spontaneously become a $V_z \neq 0$ state. Numerical results indicate that a low plasma density favors the system remaining at the $V_z=0$ state — i.e., in a state without a quasistatic magnetic field (QMF).

In conclusion, we have presented a theory for self-consistently understanding the presence of a QMF in laser-plasma interactions. The presence of a QMF is related to a transition of the system from a $V_z=0$ state to a $V_z \neq 0$ state owing to the stability requirement of the system. Stability analyses reveal that the presence of a QMF does not always take place but conditionally depends on the system parameters. Meanwhile, we also find that our theory can qualitatively explain some parameter-dependent variations of the electron spectrum observed in experiments.

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